An epistemology
(and some sketches of a related ontology)
of quantities

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Workshop: Physical quantities and measurands: Epistemological issues
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Abstract

The International Vocabulary of Metrology (VIM, JCGM 200:2012) clearly identifies quantities as the object of measurement, but then is less clear on the nature of the basic relation that through (for example) measurement connects measurands (like the length L_a of a given rod *a*) and quantity values (like 1.2345 m): is it a reasonable attribution (as the VIM suggests in def 2.1), a determination (as stated by the previous editions of the VIM), a representation (as assumed by the representational theories of measurement), ...? or isn't it rather an equality, as the customary equation L_a = 1.2345 m supposes?
An exploration of this issue leads us to better understanding the crucial role of measurand definition in a measurement process.

Context

Through (e.g.) measurement we obtain information of the sort: (*)

$L_a = 1.2345$ m (the length L_a of a given rod *a* is equal to 1.2345 m)

referring to:

- general quantities (L)
- objects (a)
- numbers (1.2345)
- units (m)

general quantities and objects: quantities of objects (L_a) (e.g., **measurands** (**))

numbers and units: values (***) of quantities (1.2345 m)

(*) Measurement uncertainty omitted.

(**) According to the VIM measurands are quantities of objects, not general quantities, not objects.

(***) ‘Value’ is intended here as in “value of a function”: axiological references are not considered.

Question

<measurand> = <value of quantity> (e.g., $L_a = 1.2345$ m)

What kind of relation is it?

- an **attribution**? (as the VIM suggests)
- a **determination**? (as stated by the previous editions of the VIM)
- a **representation**? (as assumed by the representational theories of measurement)
- ...?
- an **equality**?

measurement: “process of experimentally obtaining one or more quantity values that can **reasonably be attributed** to a quantity” [VIM3]

measurement: “set of operations having the object of **determining** a value of a quantity” [VIM2]

Purpose: domain (not fundamental) epistemo/onto-logy

… useful for reporting measurement results in a consistent and simple way

(some background assumptions:

- quantities are specific properties (*)
- individual properties are instances of general properties

- both objects and properties exist…
- … and I am agnostic as for their relations
  - objects as property-bearers?
  - properties as entities that objects possess?

)

(*) In a generalized sense of ‘property’: <shape>, and not only <is spheric>, is a property.

‘Value of a quantity’: a messy concept...

“the expression of a quantity in terms of a number and an appropriate unit of measurement” [VIM1]

“magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number” [VIM2]

“number and reference together expressing magnitude of a quantity” [VIM3]

?

‘Value of a quantity’: a commonality...

“the expression of a quantity in terms of a number and an appropriate unit of measurement” [VIM1]

“magnitude of a particular quantity generally expressed as a unit of measurement multiplied by a number” [VIM2]

“number and reference together expressing magnitude of a quantity” [VIM3]

?

ON THE MEASUREMENT OF QUANTITIES.

1.] EVERY expression of a Quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity to be expressed, which is taken as a standard of reference. The other component is the number of times the standard is to be taken in order to make up the required quantity. The standard quantity is technically called the Unit, and the number is called the Numerical Value of the quantity.

[J.C.Maxwell, A treatise on electricity and magnetism, 1873, p. 1]

Maxwell...

“Every **expression** of a Quantity consists of two factors or components. One of these is the **name** of a certain known quantity of the same kind as the quantity to be expressed, which is taken as a standard of reference. The other component is the number[(*)] of times the standard is to be taken in order to make up the required quantity. The standard quantity is technically called the Unit, and the number is called the Numerical Value of the quantity.”

Maxwell did not write about quantities but about their linguistic expression

That is, he did not explain

$Q = \{Q\}[Q]$ (e.g, $L_a = 1.2345$ m)

but

$expr(Q) =$ “$\{Q\}$”, “$[Q]$”

[(*)] Small mistake, sorry Mr. Maxwell: “The other component is the numeral designating the number...”

A (small?) consequence

“The value of a quantity is generally expressed as the product of a number and a unit.”

[BIPM, SI Brochure, 2006, 1.1]

Under the (plausible…) supposition that $expr(X) \neq X$

are values of quantities such products or *expressed as* products?

A (not so small?) consequence

"The unit is a particular example of the **quantity** [...] used as a reference."

[BIPM, SI Brochure, **2006**, 1.1]

"The unit is a particular example of the **value of a quantity** [...] used as a reference."

[BIPM, draft SI Brochure, **2016**, 2]

?

The position that units are values is supported by the following argument:

(i) since the value of a quantity is the product of a number and a unit,

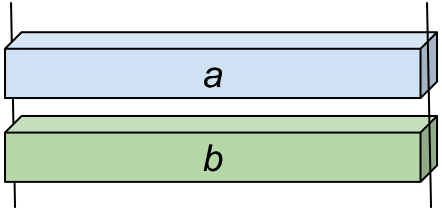
(ii) and multiplying something by a number does not change its nature,

(iii) then values and units must have the same nature;

(iv) hence units are values.

A metrology-oriented epistemo/onto-logy of quantities (1)

This experimental situation:



is usually described (*) as the claim that $L_a = L_b$

a result that does not require measuring, referring to units, introducing values, ...

(From an ontological point of view, this is compatible with:

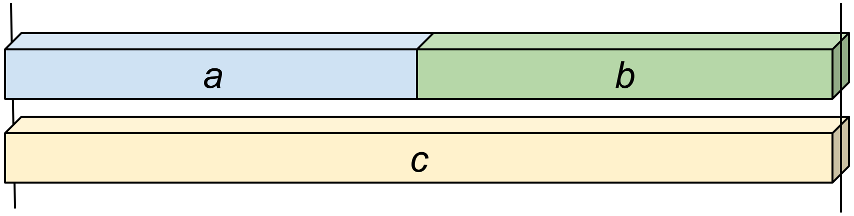
- (intensional) L_a and L_b instantiate a universal individual length L_i
- (extensional) L_a and L_b belong the same equivalence class L_i of particular individual lengths

In both cases we can refer to the entities L_i as (abstract) individual lengths)

(*) Measurement uncertainty omitted.

A metrology-oriented epistemo/onto-logy of quantities (2)

This experimental situation:



is usually described (*) as the claim that $L_c = L_a \oplus L_b$

a result that does not require measuring, referring to units, introducing values, ...

And since $L_a = L_b$, then $L_c = L_a \oplus L_a$ and finally $L_c = 2\ L_a$

a result that does not require measuring, defining units, introducing values, ...

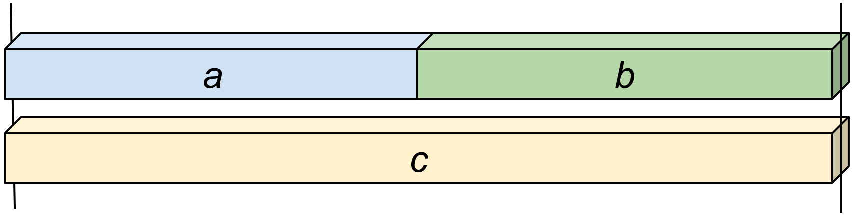
(*) Measurement uncertainty omitted.

A metrology-oriented epistemo/onto-logy of quantities (3)

Under the hypothesis that L_a is sufficiently stable and then can be given a name,

let us define $L_{ref} := L_a$

This experimental situation:



can be then described as ($L_c = L_a \oplus L_b$ or $L_c = L_a \oplus L_a$ or $L_c = 2\ L_a$ or ...)

$$L_c = 2\ L_{ref}$$

a result that does not require measuring, referring to units, introducing values, ...

A metrology-oriented epistemo/onto-logy of quantities (4)

$L_c = 2\ L_a$ is an equality of quantities of objects

$L_c = 2\ L_{ref}$ is (the same relation, and therefore also) an equality of quantities of objects

... but since we have assumed the “sufficient stability” of L_a and we have defined it as L_{ref}

then we have started to use L_a, under the name of L_{ref}, as a unit...

... so that 2 L_{ref} has become a value of quantity:

$L_c = 2\ L_a$ is a case of <quantity of object *x*> = <quantity of object *y*> i.e., $Q_x = Q_y$

$L_c = 2\ L_{ref}$ is (also) a case of <quantity of object *x*> = <value of quantity> i.e., $Q_x = \{Q\}[Q]$

A metrology-oriented epistemo/onto-logy of quantities (5)

The difference between <quantity of object *x*> = <quantity of object *y*> i.e., $Q_x = Q_y$
and <quantity of object *x*> = <value of quantity> i.e., $Q_x = \{Q\}[Q]$
is epistemological, not ontological

The equation <quantity of object *x*> = <value of quantity> i.e., $Q_x = \{Q\}[Q]$
is a claim of actual **equality** (not just representation (*))

Its meaning is:

- [**epistemological** layer]
  <quantity of object *x*> and <value of quantity> **are known** differently
- [**ontological** layer]
  and nevertheless they **are** the same entity

(*) This implies that the representational theories of measurement are generic theories of representation, and not specifically theories of measurement.

A metrology-oriented epistemo/onto-logy of quantities (6)

Again, the equation $Q_x = \{Q\}[Q]$ means:

general quantity

↑ *is an instance of*

individual quantity

is known as ↓ *is known as*

quantity of an object | value of a quantity

for example:

length, L

↑ *is an instance of*

individual length, L_i

is known as ↓ *is known as*

the length of a rod a, L_a | 1.2345 m

Intensions are different, but the extension is the same

Consequences (1)

[**epistemological** layer]

The equation $Q_x = \{Q\}[Q]$ is informative because of this difference:

- before the measurement, I knew the individual length L_x as the length of a given object *x* and I had (implicitly) built a set $\{L_i\}$ of individual lengths from a named (unit) quantity and its (non necessarily integer) multiples
- after the measurement I discovered which L_i is equal to L_x

[**ontological** layer]

The underlying ontology is simple: values of quantities are quantities

Consequences (2)

[again on the **epistemological** layer]

The equation $Q_x = \{Q\}[Q]$ includes two terms corresponding to definite descriptions:

- Q_x (e.g., the length L_a of a given rod *a*)
- $[Q]$ (e.g., the metre)

It is informative if such descriptions designate existing, sufficiently well defined entities:

$$Q_x = \ldots [Q]$$

measurand definition:
an in-context process,
explicitly or implicitly performed by
each measurer
in field conditions

unit definition:
a state-of-the-art process,
explicitly performed by
the best metrology institutions
in reference conditions

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Thank you for your kind attention

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