The (non-traditional) concept of measurement uncertainty is tied to the (traditional) concept of quantity value: analysis and consequences

Luca Mari
Università Cattaneo – LIUC, Italy

The Taming of measurement uncertainties: theoretical, practical, and philosophical issues
Université Paris Diderot, 4.12.12

“value”: a polysemic term

value: “the worth, desirability, or utility of a thing”
[Oxford Dict]

“*Homo sapiens* is also *Homo valuens*: humans are evaluative animals. We have a natural tendency to take an evaluative stance of pro or con toward virtually everything. Most of the things we see we view in a positive or negative light.”
[N.Rescher]

This is the <u>axiological side</u> of the concept

A quantity value is neither positive nor negative per se (there are no “quantity disvalues”)

Quantity values are (analogous to) values of variables, i.e., elements of sets

‘quantity value’

“[the value of a quantity is a] number and reference together expressing magnitude of a quantity”

[VIM]

“the value of a quantity is generally expressed as the product of a number and a unit”

[SI Brochure]

Let us assume the conservative version [J.C.Maxwell]:

$$q = \{q\} \cdot [q]$$

i.e.,

(individual) quantity = quantity value

i.e.,

(individual) quantity = numerical quantity value · measurement unit

and let us focus on the entity $\{q\}$, the numerical quantity value

‘numerical quantity value’

“[the numerical quantity value is a] number in the expression of a quantity value, other than any number serving as the reference”
[VIM]

the numerical quantity value is a <u>number</u>:
→ which kind of number?

- a real number?
- a natural number?
- a decimal number with a finite number of “significant digits”?
- …

and:
→ what are the reasons and the implications of this assumption?

(note, e.g.: numerical quantity values for ordinal quantities could be ordered identifiers, which are not necessarily numbers)

Why this emphasis on numbers?

“numbers are in the world” [Kepler]
the world “is written in mathematical characters” [Galileo]

and then:

- in the deterministic case, physical quantities have been traditionally modeled as continuously-varying variables, and therefore their values as real numbers (*physical laws as differential equations*)

- in the stochastic case, “a random variable is a number $x(z)$ assigned to every outcome of an experiment z” [A.Papoulis], and the problem is to estimate the distribution parameters (*true values of quantities as expected values of distributions*)

It is a traditional, widely accepted, convenient position...
... and nevertheless, that “numbers are in the world”
is **metaphysical hypothesis**

“a random variable is a number $x(z)$ assigned to every outcome of an experiment z” [A.Papoulis]

→ are there any criteria for such an assignment?
… maybe it must be a homomorphism, i.e., number relations “reflect” the structure of the experiment?

and the problem is to estimate the distribution parameters
(*true values of quantities as expected values of distributions*)

→ so that distribution parameters convey relevant information on the experiment

An ongoing transition?
[operative side: measured quantity value]

VIM2 (and GUM) [1993]

<u>measurement</u>
set of operations having the object of determining <u>a</u> value of a quantity

<u>result of a measurement</u>
<u>value</u> attributed to a measurand, obtained by measurement

VIM3 [2007]

<u>measurement</u>
process of experimentally obtaining <u>one or more</u> quantity values that can reasonably be attributed to a quantity

<u>measurement result</u>
<u>set of quantity values</u> being attributed to a measurand together with any other available relevant information

An ongoing transition?
[ontological side: measurand value]

“When the definitional uncertainty associated with the measurand is considered to be negligible compared to the other components of the measurement uncertainty, the measurand may be considered to have an ‘essentially unique’ true quantity value. This is the approach taken by the GUM.”

[VIM]

But what would the VIM3 say when definitional uncertainty is not negligible?

That the measurand has many (infinite...) values?

“[a coverage interval is the] interval containing the set of true quantity values of a measurand with a stated probability, based on the information available”

[VIM]

The VIM3 started a dangerous navigation,
and it is now half way (?) to the target

(whereas the GUM remained in the safe, good old harbor...)

Uncertainty of what?

1. “[If] the error in the measurement result is small [...] the uncertainty of a result of a measurement is [...] an indication of the likelihood that the measurement result is near the value of the measurand.”

2. “[Otherwise] it is simply an estimate of the likelihood of nearness to the best value that is consistent with presently available knowledge.”

[GUM]

“[If] no significant systematic effects have been overlooked, one can assume that the measurement result is a reliable estimate of the value of the measurand and that <u>its combined standard uncertainty is a reliable measure of its possible error</u>.”

[GUM]

A different view

“It is *we* who assign numbers to nature. The phenomena themselves exhibit only qualities we observe. Everything numerical [...] is brought in by ourselves when we devise procedures for measurement.” [R.Carnap]

According to this position, the role of numbers in quantity values is not <u>metaphysical</u>, but <u>informational</u>:

numbers are not <u>in</u> the world, but can be used <u>to represent</u> it

quantity values are not <u>inherent of</u> quantities,
but are <u>assigned to</u> them

quantities do not <u>have</u> values, but <u>are represented</u> by values

A simpler, less metaphysics laden,
more informationally oriented standpoint
can be adopted...

... if only the concept of quantity value
is reconsidered

‘Quantity value’ again
[ontological side]

Quantities do not have values in themselves:

asking whether a measured quantity value is near
the value of the measurand is just meaningless

‘Quantity value’ again
[operative side]

Back to:
quantity values are (analogous to) values of variables,
i.e., they are elements of sets

Indeed, sets can be sets of numbers, but also of intervals, PDFs, …

so that a quantity value might be a number but also an interval,
a PDF, … (always together with the quantity unit, of course)

<u>A (measured) quantity value is the entity</u>
(number, interval, PDF, ... with a quantity unit)
<u>which represents the measurand</u>
<u>on the basis on the available information</u>

‘Measurement result’ again

A measurement result must faithfully represent the information that has been acquired by means of experiment

The same information is compatible with multiple representations

In the simplest case of quantity values represented as intervals, e.g.:

[9.95, 10.05]	[9, 11]
more *specific* information	less *specific* information
less *reliable* information	more *reliable* information

[0, 100000000]
VERY unspecific but COMPLETELY reliable

<u>There is a trade-off between *specificity* and *reliability*</u>

(and the only way the increase one without decreasing the other one is to perform a better experiment)

‘Uncertainty’ again

While *specificity* is a property of quantity values
([9.95, 10.05] is more specific than [9, 11])

reliability describes...
...the degree of certainty attributed to a quantity value
on the basis of the available information

Indeed, when in the rest of the world (...!) it is said that
“probability is the logic of uncertainty” the concept is:
$P(A)=1 \rightarrow$ A is certain / necessary
$P(A)=0 \rightarrow$ A is impossible

that is, (un)reliability = (un)certainty, ranging in [0, 1]

whereas in metrology (un)specificity ~ (un)certainty, ranging in [0, ∞]

Numbers again

Of course, this does not imply that we have lost numbers:

were this required, a quantity value (interval, PDF, fuzzy set, …) might be represented by a number

The (non-traditional) concept of measurement uncertainty
is tied to the (traditional) concept of quantity value:
analysis and consequences

Thank you for the kind attention

Luca Mari

lmari@liuc.it