(Some notes about)
Truth in measurement

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Structure of the presentation

Introduction
A model of measurement:
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simple measurement
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Analysis of truth conditions

My context

(terminology as a point of contact between engineering and philosophy)

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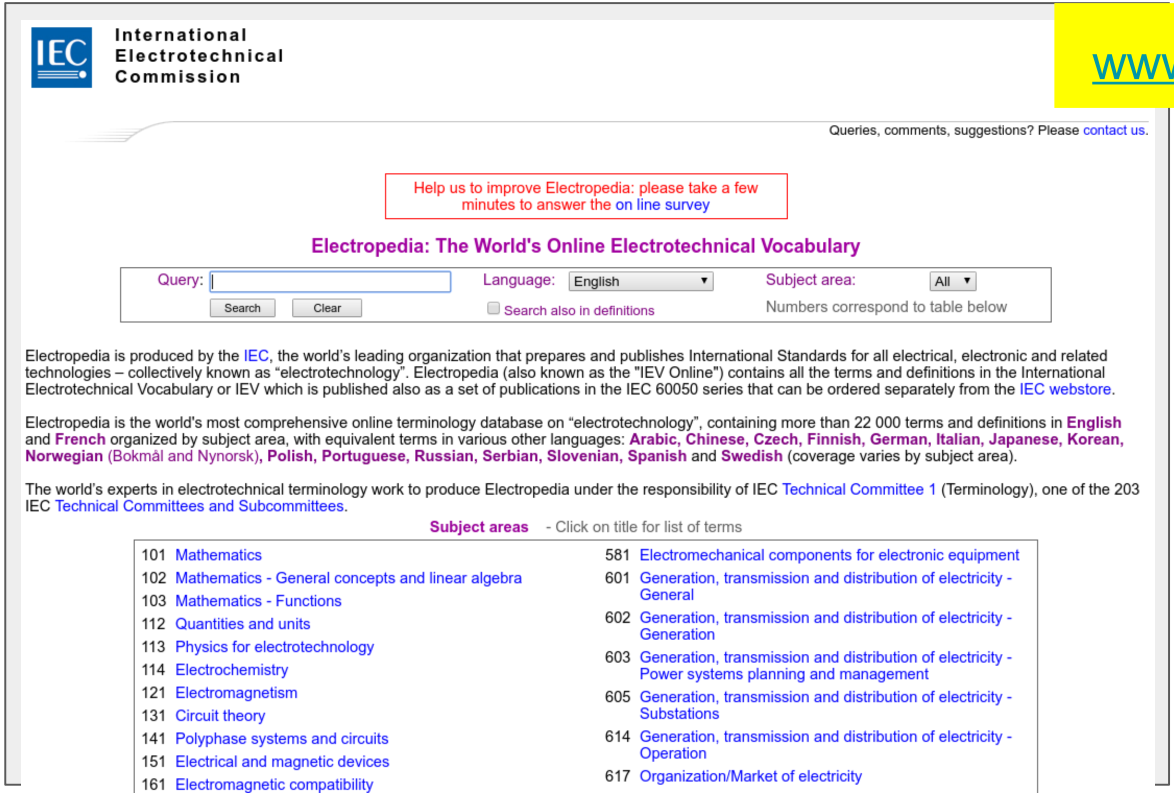

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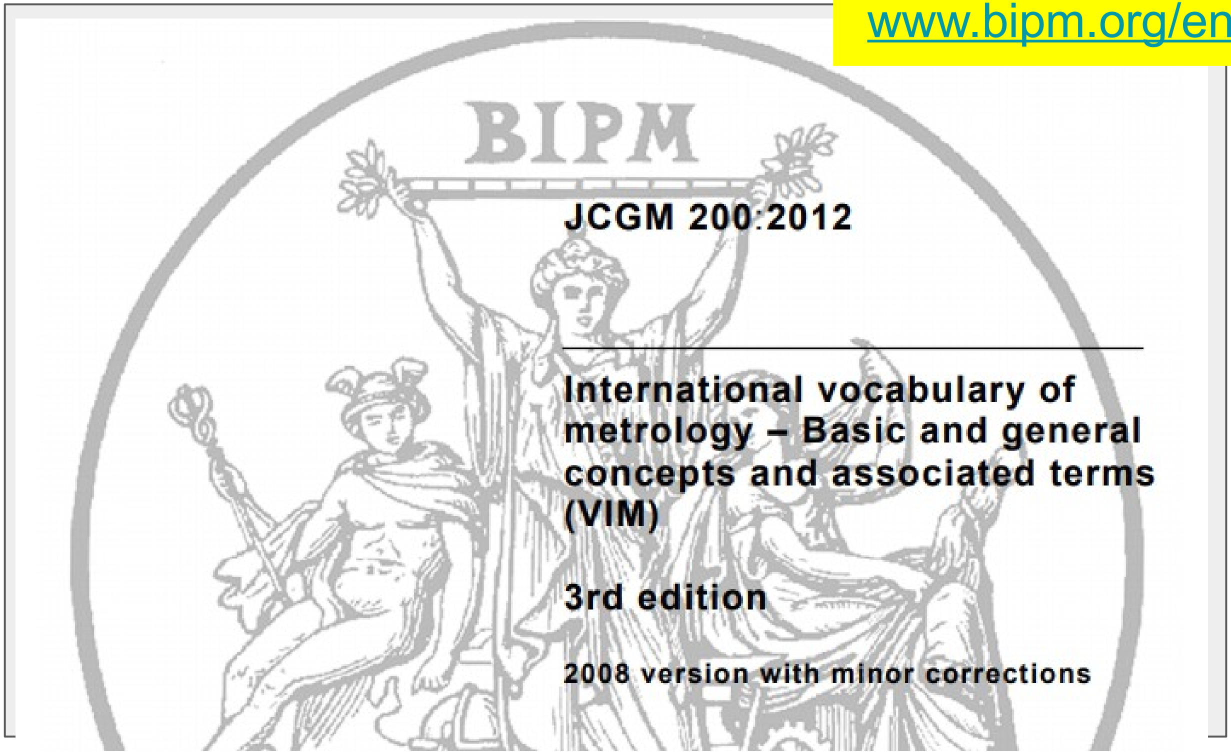
Subject areas - Click on title for list of terms

101	Mathematics	581	Electromechanical components for electronic equipment
102	Mathematics - General concepts and linear algebra	601	Generation, transmission and distribution of electricity - General
103	Mathematics - Functions	602	Generation, transmission and distribution of electricity - Generation
112	Quantities and units	603	Generation, transmission and distribution of electricity - Power systems planning and management
113	Physics for electrotechnology	605	Generation, transmission and distribution of electricity - Substations
114	Electrochemistry	614	Generation, transmission and distribution of electricity - Operation
121	Electromagnetism	617	Organization/Market of electricity
131	Circuit theory		
141	Polyphase systems and circuits		
151	Electrical and magnetic devices		
161	Electromagnetic compatibility		

My context
(terminology as a point of contact between engineering and philosophy)

www.bipm.org/en/publications/guides/vim.html


BIPM

JCGM 200:2012

International vocabulary of metrology – Basic and general concepts and associated terms (VIM)

3rd edition

2008 version with minor corrections

Introduction

(the problem)

True instruments?

“When I place two arbitrary bodies on the pans of a **true balance**, the balance will generally not be in equilibrium, but one pan will sink. Exceptionally, I shall find certain pairs of bodies *a* and *b* which, when placed on the balance, will not disturb its equilibrium.” (H. Helmholtz, 1887, Engl. transl. 1977, p.91)

Is truth a feature of measuring instruments?

Other translations for the German term “richtigen” use, e.g., “correct” (1930, p.19)

A measuring instrument is a designed entity: a true / correct / accurate instrument behaves as expected by design

Black box modeling

A measuring instrument can be modeled as a black box

The usual understanding:
“The particular quantity to be measured is called a measurand. Its **(true) value** is the result that would be obtained by a **perfect measurement**. Since perfect measurements are only imaginary, a true value is always indeterminate and unknown.” (P.P.L. Regtien, Measurement science for engineers, 2004, p.44)

Is truth a feature of values of quantities?

Black box modeling /2

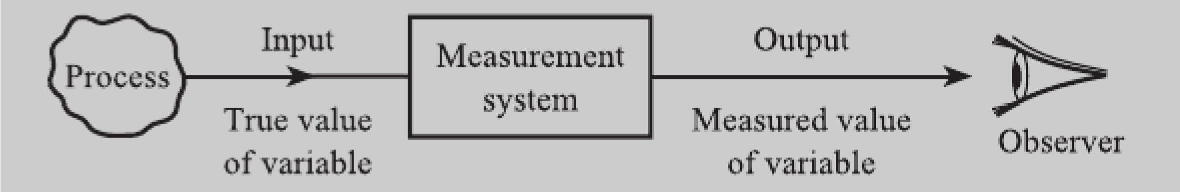
Hence the black box model is

where the measured value is an estimate of the true value of the measurand

The underlying principle is well known from statistics:
the sample means m_i converge to the population mean $\boldsymbol{\mu}$,
i.e., the (usually unknown) value $\boldsymbol{\mu}$ is estimated by the experimental values m_i

Black box modeling /3

The extreme version (from J.P. Bentley, Principles of measurement systems 2005[4], p.3)



– Is the input to a measuring instrument really a value? (instead of the measurand)
– Are these the values of variables? (instead of quantities)
– Are values of variables observable?

(this might be interpreted as a “transmission model” of measurement)

Something needs to be better understood...

True values of what?

A search in the Library of Congress catalog (the phrase “true value” in keywords) shows that the term has several different meanings,
e.g., “realize your true value and pursue your passions”, “the true value of friendship”, “the true value of experience in medicine”, “true value of Pi”

This is rooted in the polysemy of “value”:
we mean by “value” an element of a set, chosen as the range of a function
(if $f: X \to Y$ and $y \in Y$, then y is a value of f)

(true) values of empirical quantities of objects

Empirical quantities can be modeled as functions from objects to values

This seems to be the implied meaning of the basic equation

(*) $Q_a = q$

e.g., $\text{Length}_{\text{this pen}} = 0.123$ m

Values are, per se, neither true nor false:

“true value” is just a shorthand for “value in an equation that is true (*)”

In this perspective,

truth in measurement is about the conditions of truth of equation (*)

We use the notation “Q_a”, instead of “$Q(a)$”, to emphasize that Q is not a function, but can be modeled as a function.

Two extreme positions

[“classical” position]
Measurement is a **determination** of values of independently existing quantities
(were the empirical process ideal, equation (*) would be **true**)

[“representational” position]
Measurement is the **assignment** of values to quantities to be represented
(were the empirical process ideal, equation (*) would be **consistent**)

Truth seems to be mainly related to the “classical” position

(and in fact dealing with truth in measurement is not so fashionable these days…)

Truth and consistency: mixing positions?

According to the International Vocabulary of Metrology (VIM),
a true value is a “quantity value consistent with the definition of a quantity”...
(jcgm.bipm.org/vim/en/2.11.html)

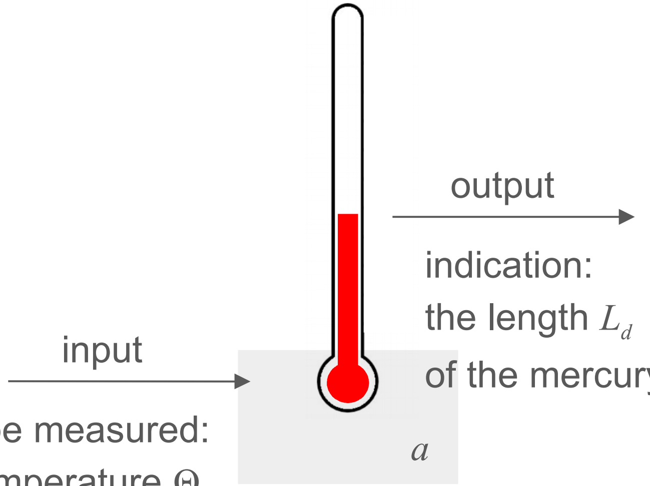
Has truth become consistency?

A model of measurement / 1

(simple pre-measurement)

Our strategy

We seek conditions of truth in the structure of a typical measurement process, based on a sensor that transduces the quantity to be measured to an indication



This is an empirical transduction:
both input and output are physical states;
values of quantities are not here yet

A bottom-up presentation, from a simple example and in an ordinal case

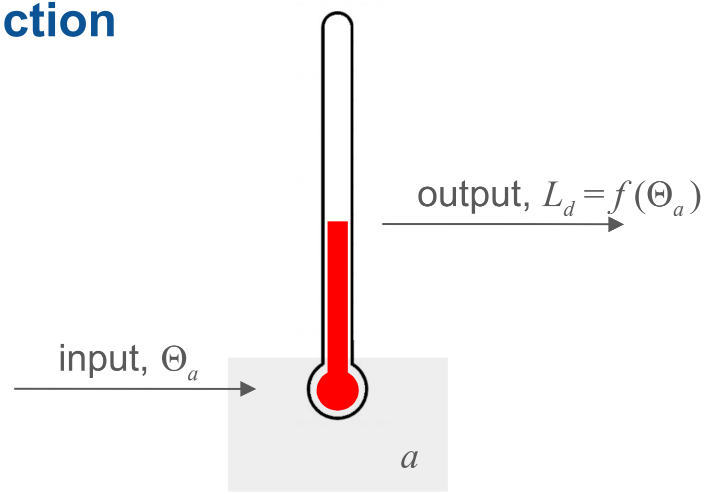
The underlying assumption: causality

The length L_d is caused by the temperature Θ_a
and, *ceteris paribus*, Θ_a is the only cause of L_d

By assuming the stability of the transduction effect,
this can be formalized as a **transduction function**

$$L_d = f(\Theta_a)$$

$$\Theta_a \xrightarrow{\quad f \quad} L_d$$



Marking and identifying lengths in the instrument

Instrument indications are lengths of the mercury in the capillary of the thermometer

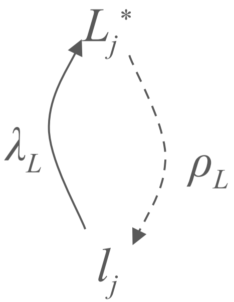
The instrument is designed so that some of these lengths can be identified by

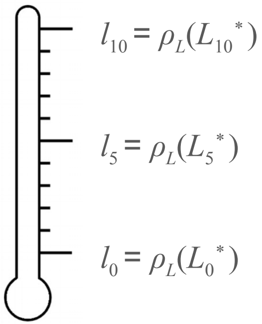
- marking a set $\{L_j^*\}$ of them, where each mark is a *private standard* of length
- assigning an identifier, l_j, to each mark position, via an injective **labeling function**

$$L_j^* = \lambda_L(l_j)$$

- inverting λ_L to identify each mark position, via a **recognition function** $\rho_L = \lambda_L^{-1}$

$$l_j = \rho_L(L_j^*)$$





Under the hypothesis that the instrument is stable, the identifiers l_j make the comparison of results of transductions performed in different times possible

Matching indications to mark positions

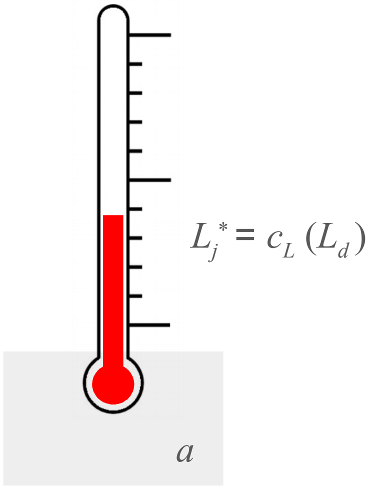
The instrument is designed so that to each instrument indication L_d

a mark position L_j^* can be associated (typically a quantization process)

This can be formalized as a **matching function**

$$L_j^* = c_L\,(L_d)$$

$$L_d \xrightarrow{\;c_L\;} L_j^*$$



where the set $\{L_j^*\}$ may be operated

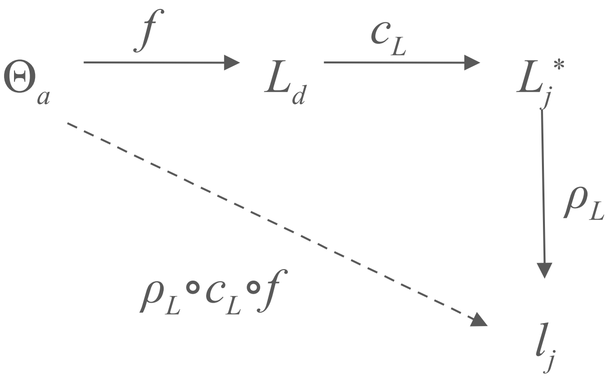
as a *private* (because *instrument-specific*) scale:

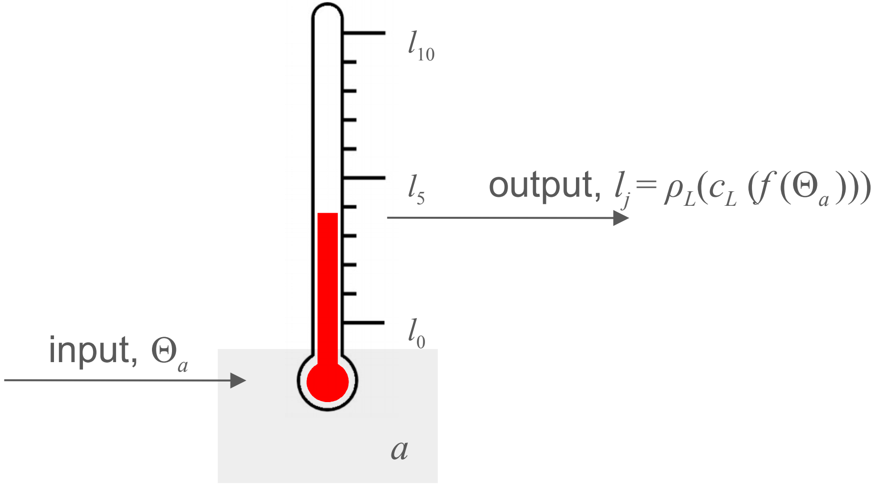
let us define $\{L_j^*\} := {}^\circ\mathrm{mydev}$

(simple) pre-measurement

By composing these maps, we obtain what we call a **pre-measurement**

$$l_j = \rho_L(c_L(f(\Theta_a)))$$





It is a sort of *private* (because *instrument-specific*) measurement, whose result is

The value of Θ_a is l_j in the scale °mydev if and only if $\rho_L(c_L(f(\Theta_a))) = l_j$

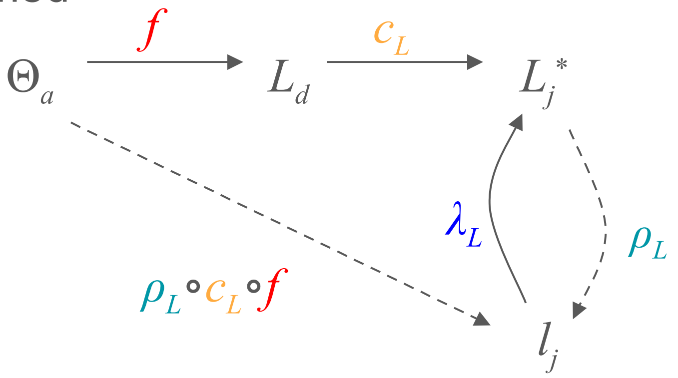
*Components of (simple) pre-measurement

Let us analyze the components of pre-measurement:

- the mappings f and c_L are empirical and need to be **modeled**
- the mapping λ_L is defined **by convention**
- the mapping ρ_L is correctly λ_L^{-1} under the hypothesis that instrument is **stable**

and on this basis for any given object a

the **transduction** $L_d = f(\Theta_a)$, the **matching** $L_j^* = c_L(L_d)$, and the **recognition** $l_j = \rho_L(L_j)$ are performed



According to this simple model, pre-measurement results may convey object-related (*objective*) information

A model of measurement / 2

(simple measurement)

From pre-measurement to measurement

Pre-measurement results are non-transferable, because private / instrument-specific

Measurement aims at producing information which is not only object-related,
but also subject-independent (“intersubjective”),
and therefore public and instrument-independent

In the tradition of physical measurement this is the task of **metrological systems**,
i.e., measurement standards mutually connected in traceability chains via calibration

Choosing and identifying temperatures

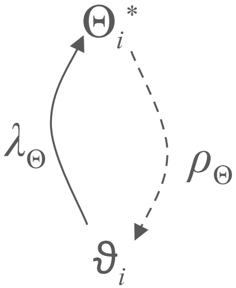
Some objects / phenomena are chosen whose temperatures are sufficiently stable, so that these temperatures can be identified by

- choosing a set $\{\Theta_i^*\}$ of them, each object being a (*public*) *standard* of temperature
- assigning an identifier, ϑ_i, to each temperature, via an injective **labeling function**

$$\Theta_i^* = \lambda_\Theta(\vartheta_i)$$

- inverting λ_Θ to identify each temperature, via a **recognition function** $\rho_\Theta = \lambda_\Theta^{-1}$

$$\vartheta_i = \rho_\Theta(\Theta_i^*)$$



...

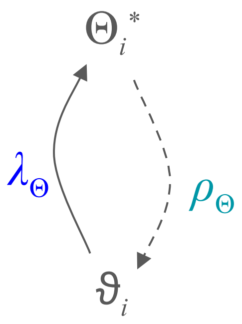
$\vartheta_0 = \rho_\Theta(\Theta_0^*)$

The set $\{\Theta_i^*\}$ may be operated as a (public) scale of temperature: let us define $\{\Theta_i^*\} :=$ °pub

*Components of (public) scale construction

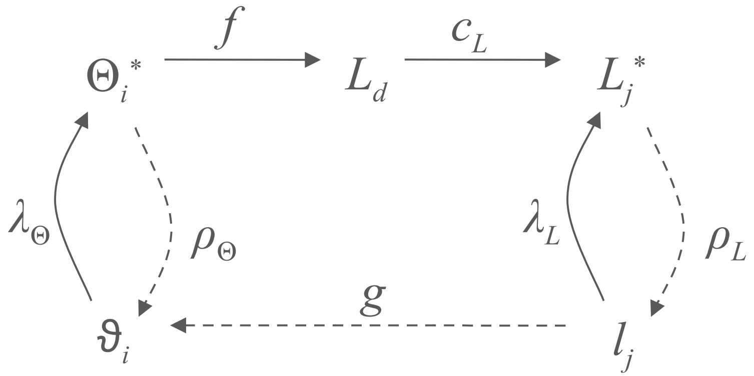
Let us analyze the components of scale construction:

- the mapping λ_Θ is defined **by convention**
- the mapping ρ_Θ is correctly λ_Θ^{-1} under the hypothesis that standards are **stable**



Calibrating instruments

After having constructed the scale of temperature °pub,
the instrument is calibrated by making it interact with the elements of °pub



$$\vartheta_i = g\,(l_j)$$

i.e.,

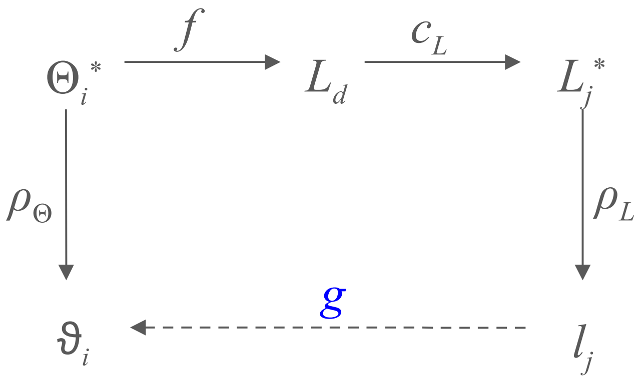
$$\rho_\Theta(\Theta_i^*) = g\,(\rho_L(c_L\,(f\,(\Theta_i^*))))$$

The result is the (extensional) definition of the **calibration function** g
as a set $\{\langle l_j, \vartheta_i\rangle\}$ of pairs ⟨private identifier, public identifier⟩

*Components of instrument calibration

Let us analyze the components of instrument calibration:

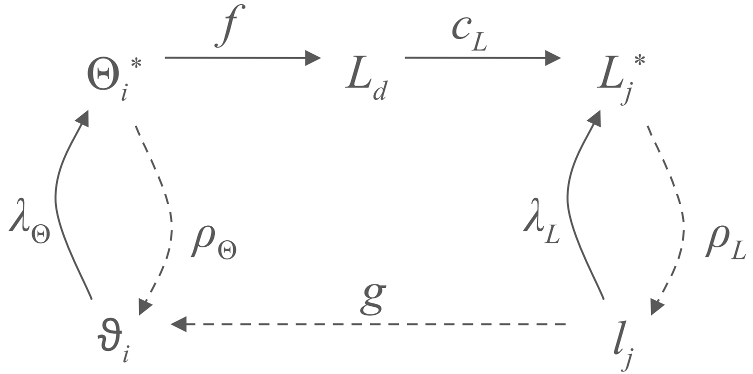
- *all the components of scale construction*
- the mapping g is correctly the set of pairs $\langle l_j, \vartheta_i \rangle = \langle (\rho_L(c_L(f(\Theta_i^*))), \rho_\Theta(\Theta_i^*) \rangle$ under the hypothesis that the instrument is **stable**



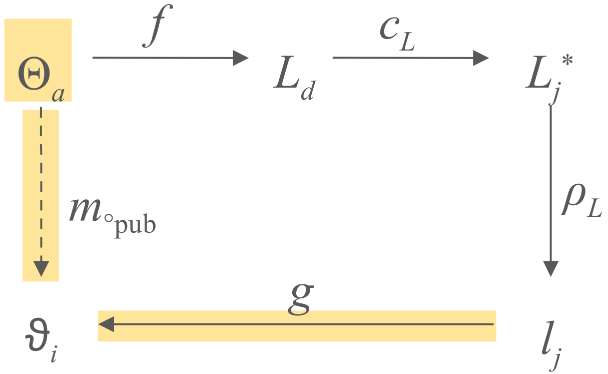
(simple) measurement

By making the calibrated instrument interact with the quantity to be measured Θ_a, we obtain a **measurement**

$$\vartheta_i = g\,(\rho_L(c_L\,(f\,(\Theta_a\,))))$$



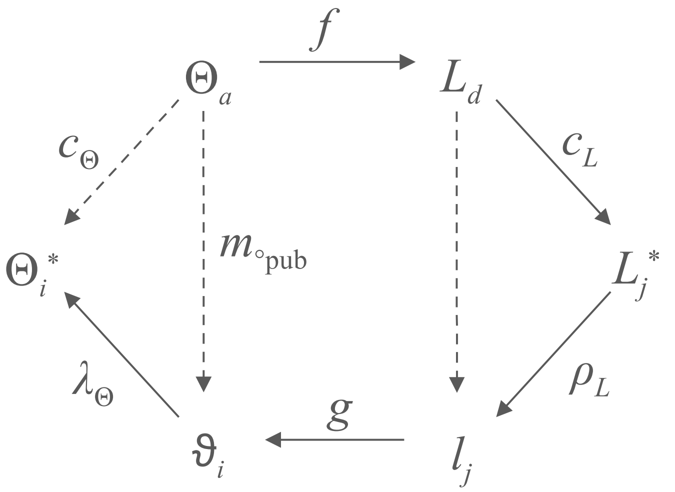
scale construction and calibration



measurement

The main differences are highlighted

The structure of (simple) measurement



The result of measurement is

$\Theta_a = \vartheta_i$ °pub (e.g., $\Theta_a = 20$ °C)

i.e.,

$c_\Theta(\Theta_a) = \Theta_i^*$ (e.g., value-of$(\Theta_a) = 20$ °C)

i.e.,

$m_{\circ\text{pub}}(\Theta_a) = \vartheta_i$ (e.g., value-in-°C-of$(\Theta_a) = 20$)

if and only if

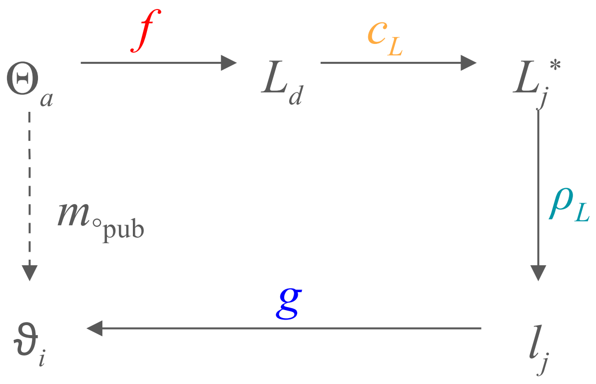
$\Theta_i^* = \lambda_\Theta(\vartheta_i) = \lambda_\Theta(g(\rho_L(c_L(f(\Theta_a)))))$

*Components of (simple) measurement

Let us analyze the components of measurement:

- *all the components of pre-measurement*
- *all the components of instrument calibration (and therefore also of scale construction)*

and on this basis for any given object a and any given calibrated instrument
the **transduction** $L_d = f(\Theta_a)$, the **matching** $L_j^* = c_L(L_d)$, the **recognition** $l_j = \rho_L(L_j)$,
and the **calibration** $\vartheta_i = g(l_j)$ are performed



According to this simple model,
(i) measurement = pre-measurement + public scale construction + instrument calibration
(ii) measurement results may convey both object-related (*objective*) and subject-independent (*intersubjective*) information

A model of measurement / 3 – hints

(less simple measurement)

From simple to more realistic measurement

We have assumed so far that

1. the sensor is perfectly selective, i.e., the transduction function f only depends on Θ
2. the quantity to which the value resulting from the measurement is attributed and the quantity that is transduced by the sensor are the same

Both these assumptions may be relaxed, thus making our model more realistic

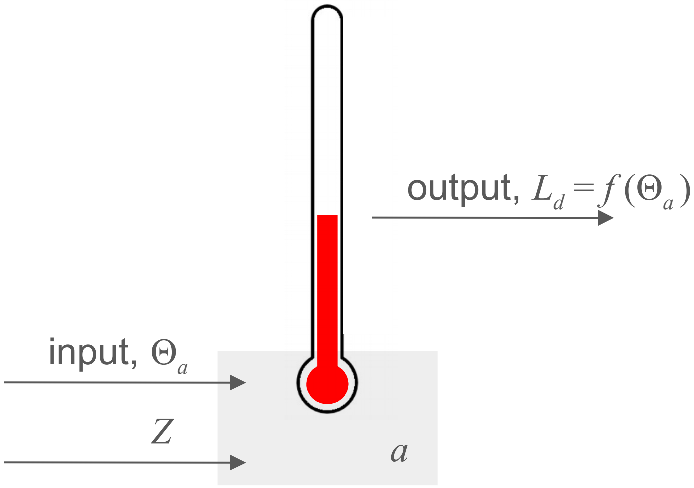
Non-perfectly selective instruments

In general, the transduced length L_d is caused not only by the temperature Θ_a but also by other (influence) quantities Z

Hence the transduction function is

$$L_d = f_Z(\Theta_a)$$

$$\Theta_a \xrightarrow{f_Z} L_d$$



The limited selectivity of the instrument reduces the objectivity of the information produced by the measurement

Different intended and effective quantities

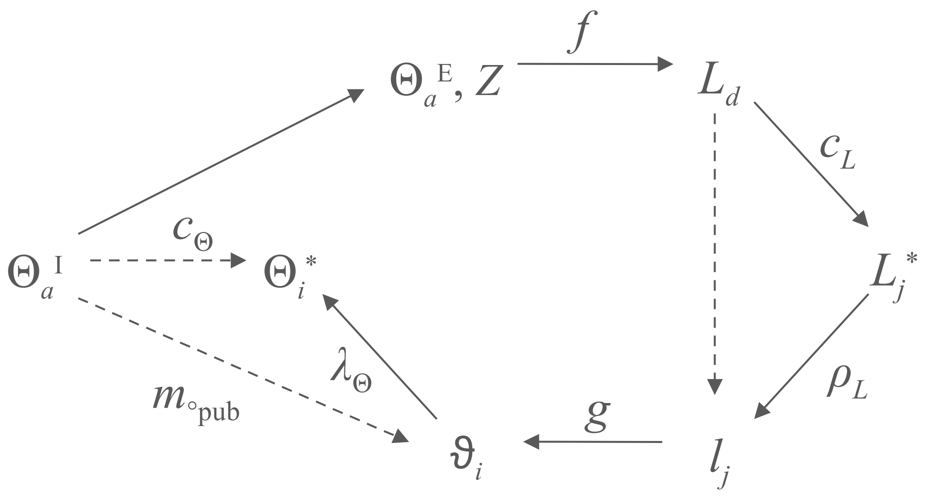
According to the International Vocabulary of Metrology (VIM),
the measurand is the “quantity intended to be measured”
(jcgm.bipm.org/vim/en/2.3.html)

In general, the **intended** quantity, to which the value is attributed,
can be defined as different from the **effective** quantity, i.e., the transduced quantity

$$\Theta_a^{\mathrm{E}} \xrightarrow{\;f\;} L_d$$

$$\Theta_a^{\mathrm{I}} \xrightarrow{\;m_{\circ\mathrm{pub}}\;} \vartheta_i$$

The structure of (less simple) measurement



The result of measurement is

$$\Theta_a^{\mathrm{I}} = \vartheta_i \circ\mathrm{pub}$$

i.e.,

$$c_\Theta(\Theta_a^{\mathrm{I}}) = \Theta_i^*$$

i.e.,

$$m_{\circ\mathrm{pub}}(\Theta_a^{\mathrm{I}}) = \vartheta_i$$

if and only if

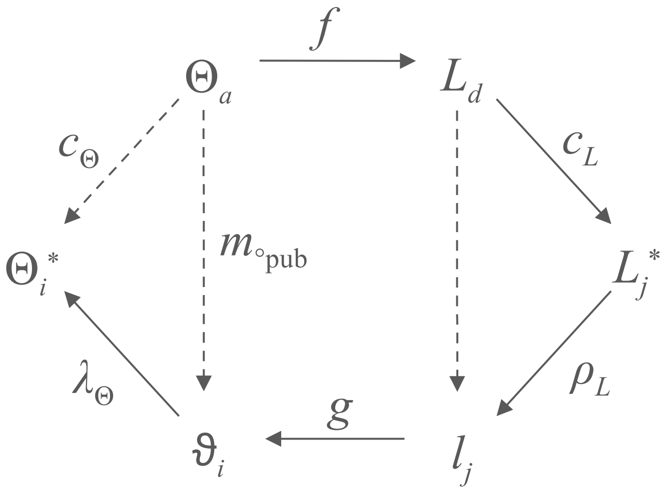
$$\Theta_i^* = \lambda_\Theta(\vartheta_i) = \lambda_\Theta(g\,(\rho_L(c_L\,(f\,(\Theta_a^{\mathrm{E}})))))$$

and (e.g.,)

$$\Theta_a^{\mathrm{E}} \approx \Theta_a^{\mathrm{I}}$$

Analysis of truth conditions

Truth conditions



Each arrow represents a function,
and each function represents a process
and is associated with a proposition with a truth value

(we say "truth of f" as a shorthand for
"truth of the proposition that $f(x) = y$")

Note that
(1) the truth conditions of a function $f: \mathrm{X} \to \mathrm{Y}$, $f(x) = y$, do not depend on the nature of f
(2) the truth conditions of a composed function $f_2 \circ f_1$ depend on the truth conditions
of f_1 and f_2, so that only the non-composed functions need to be considered

Truth conditions /2

- the process represented by f exists
- the process represented by f is stable

[on the process]

$$\mathrm{X} \xrightarrow{f} \mathrm{Y}$$

[on the domain X]

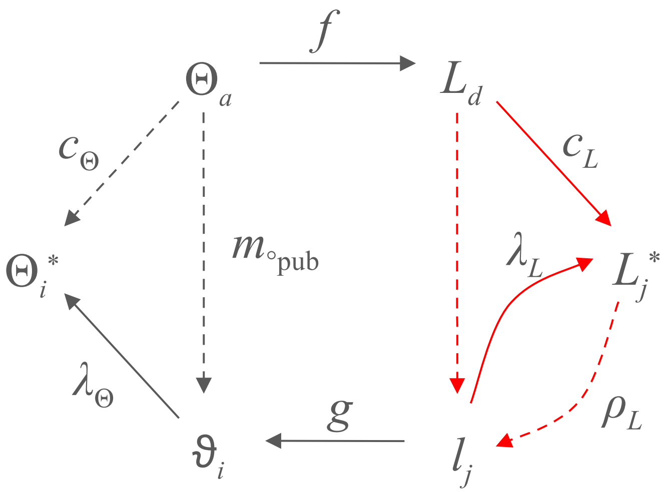
- X exists
- the structure of X is as hypothesized
- the structure of X is stable

[on the range Y]

- Y exists
- the structure of Y is as hypothesized
- the structure of Y is stable

If these conditions are satisfied, then "$f(x) = y$" is true if and only if the process represented by f produces the entity represented by y when it receives the entity represented by x as input

Analysis of truth conditions

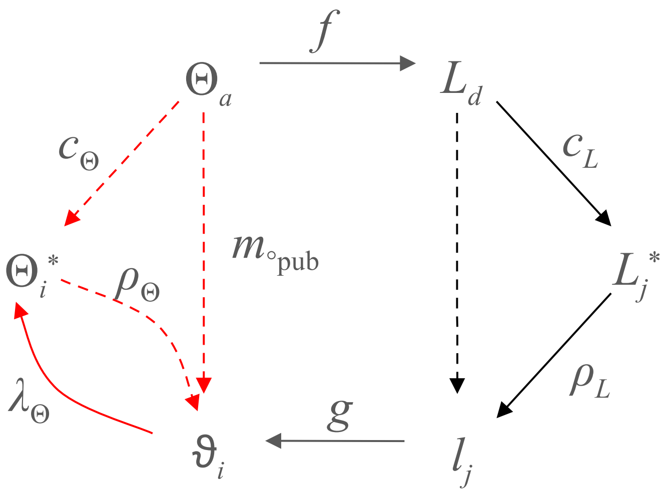


1. The existence and structure of the domain and range of λ_L and ρ_L are non-problematic:
   $\{l_j\}$ is chosen in correspondence with $\{L_j^*\}$

2. The existence and structure of the domain and range of c_L are non-problematic:
   the set $\{L_j^*\}$ is constructed and so it exists

3. The stability of $\{L_j^*\}$ depends on
   the stability of the instrument

Analysis of truth conditions /2

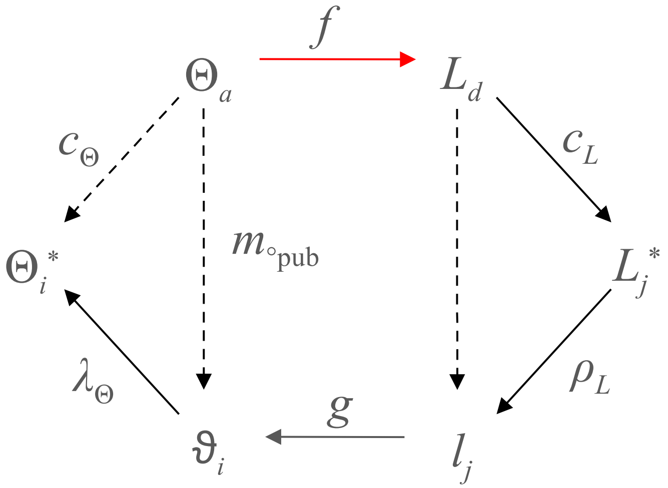


4. The existence and structure of the domain and range of λ_Θ and ρ_Θ are non-problematic:
   $\{\vartheta_i\}$ is chosen in correspondence with $\{\Theta_i^*\}$

5. The existence and structure of the domain and range of c_Θ are non-problematic:
   the set $\{\Theta_i^*\}$ is constructed and so it exists

6. The stability of $\{\Theta_i^*\}$ depends on the stability of the chosen standards

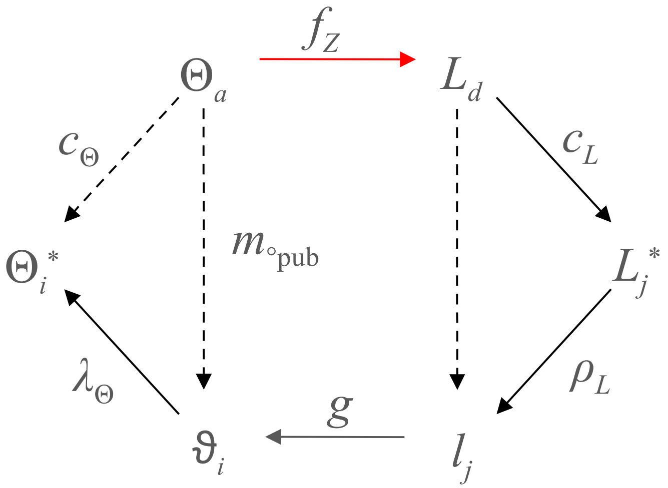
Analysis of truth conditions /3



7. The existence and structure of the domain of f is model-dependent: the transduction effect is assumed to be many-to-one and order-reflecting

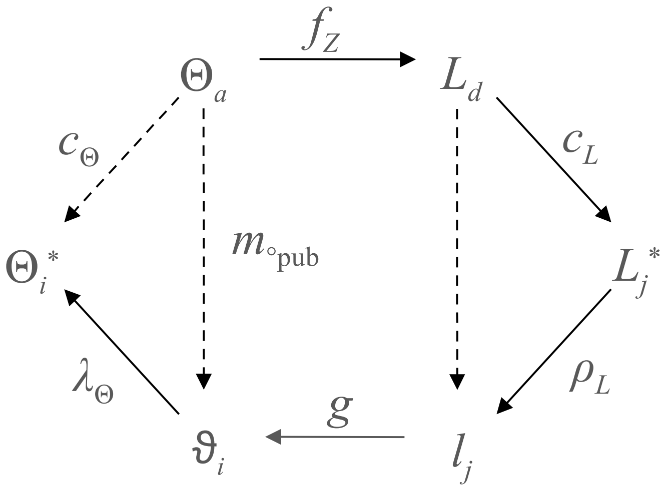
   The structure of Θ is inferred, given the transduction effect, from the structure of *L*

Analysis of truth conditions /4



7[BIS]. Finally, if the instrument is not perfectly selective, so that influence quantities Z are to be considered, and f_Z represents the transduction in conditions Z, the definition of f_Z depends on the information available on the influence quantities

Preliminary conclusions



This schema is a powerful tool for analysing the truth conditions of propositions stating measurement results. In the light of it, four conclusions seem to be justified:

(1) a proposition like $\Theta_i^* = \lambda_\Theta(\vartheta_i) = \lambda_\Theta(g\,(\rho_L(c_L\,(f_Z\,(\Theta_a)))))$ can be **true**

(2) the quantity Θ_i^* is the **true value** of Θ_a, provided that proposition (1) is true

(3) the truth of (1) **depends on the stability** of the instrument and of the chosen standards **and the model** f of the transduction

(4) propositions (1) and (2), even though model-dependent, are **about the world, not about a model**

Thank you

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