(Some notes about) Truth in measurement

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(terminology as a point of contact between engineering and philosophy)
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Introduction

(the problem)
True instruments?

“When I place two arbitrary bodies on the pans of a true balance, the balance will generally not be in equilibrium, but one pan will sink. Exceptionally, I shall find certain pairs of bodies $a$ and $b$ which, when placed on the balance, will not disturb its equilibrium.” (H. Helmholtz, 1887, Engl. transl. 1977, p.91)

**Is truth a feature of measuring instruments?**

Other translations for the German term “richtigen” use, e.g., “correct” (1930, p.19)

A measuring instrument is a designed entity: a true / correct / accurate instrument behaves as expected by design
Black box modeling

A measuring instrument can be modeled as a black box

The usual understanding:
“The particular quantity to be measured is called a measurand. Its (true) value is the result that would be obtained by a perfect measurement. Since perfect measurements are only imaginary, a true value is always indeterminate and unknown.” (P.P.L. Regtien, Measurement science for engineers, 2004, p.44)

Is truth a feature of values of quantities?
Black box modeling /2

Hence the black box model is

where the measured value is an estimate of the true value of the measurand

The underlying principle is well known from statistics: the sample means $m_i$ converge to the population mean $\mu$, i.e., the (usually unknown) value $\mu$ is estimated by the experimental values $m_i$. 
Black box modeling /3

The extreme version (from J.P. Bentley, Principles of measurement systems 2005⁴, p.3)

– Is the input to a measuring instrument really a value? (instead of the measurand)
– Are these the values of variables? (instead of quantities)
– Are values of variables observable?

(this might be interpreted as a “transmission model” of measurement)

Something needs to be better understood...
True values of what?

A search in the Library of Congress catalog (the phrase “true value” in keywords) shows that the term has several different meanings, e.g., “realize your true value and pursue your passions”, “the true value of friendship”, “the true value of experience in medicine”, “true value of Pi”

This is rooted in the polysemy of “value”: we mean by “value” an element of a set, chosen as the range of a function (if \( f : X \rightarrow Y \) and \( y \in Y \), then \( y \) is a value of \( f \))
(true) values of empirical quantities of objects

Empirical quantities can be modeled as functions from objects to values.

This seems to be the implied meaning of the basic equation

(*) \( Q_a = q \)

e.g., Length\text{this pen} = 0.123 \text{ m}

Values are, per se, neither true nor false:
“true value” is just a shorthand for “value in an equation that is true (*)”

In this perspective,

truth in measurement is about the conditions of truth of equation (*)

We use the notation “\( Q_a \)”, instead of “\( Q(a) \)”, to emphasize that \( Q \) is not a function, but can be modeled as a function.
Two extreme positions

[“classical” position]
Measurement is a **determination** of values of independently existing quantities (were the empirical process ideal, equation (*) would be **true**)

[“representational” position]
Measurement is the **assignment** of values to quantities to be represented (were the empirical process ideal, equation (*) would be **consistent**)

**Truth seems to be mainly related to the “classical” position**

(and in fact dealing with truth in measurement is not so fashionable these days…)
Truth and consistency: mixing positions?

According to the International Vocabulary of Metrology (VIM), a true value is a “quantity value consistent with the definition of a quantity”...

(jcgm.bipm.org/vim/en/2.11.html)

Has truth become consistency?
A model of measurement / 1

(simple pre-measurement)
Our strategy

We seek conditions of truth in the structure of a typical measurement process, based on a sensor that transduces the quantity to be measured to an indication. This is an empirical transduction: both input and output are physical states; values of quantities are not here yet.

A bottom-up presentation, from a simple example and in an ordinal case.
The underlying assumption: causality

The length $L_d$ is caused by the temperature $\Theta_a$ and, ceteris paribus, $\Theta_a$ is the only cause of $L_d$.

By assuming the stability of the transduction effect, this can be formalized as a **transduction function**

\[ L_d = f(\Theta_a) \]

![Diagram showing the transduction process](image)
Marking and identifying lengths in the instrument

Instrument indications are lengths of the mercury in the capillary of the thermometer. The instrument is designed so that some of these lengths can be identified by:

- marking a set \( \{L_j^*\} \) of them, where each mark is a *private standard* of length
- assigning an identifier, \( l_j \), to each mark position, via an injective *labeling function* \( L_j^* = \lambda_L(l_j) \)
- inverting \( \lambda_L \) to identify each mark position, via a *recognition function* \( \rho_L = \lambda_L^{-1} \)

\[ l_j = \rho_L(L_j^*) \]

Under the hypothesis that the instrument is stable, the identifiers \( l_j \) make the comparison of results of transductions performed in different times possible.
Matching indications to mark positions

The instrument is designed so that to each instrument indication $L_d$ a mark position $L_j^*$ can be associated (typically a quantization process). This can be formalized as a matching function

$$L_j^* = c_L (L_d)$$

where the set $\{L_j^*\}$ may be operated as a private (because instrument-specific) scale: let us define $\{L_j^*\} := °\text{mydev}$
(simple) pre-measurement

By composing these maps, we obtain what we call a pre-measurement

\[ l_j = \rho_L(c_L(f(\Theta_a))) \]

\[ \Theta_a \xrightarrow{f} L_d \xrightarrow{c_L} L^*_j \]

\[ \rho_L \circ c_L \circ f \]

\[ \rho_L \]

\[ l_j \]

It is a sort of private (because instrument-specific) measurement, whose result is

The value of \( \Theta_a \) is \( l_j \) in the scale \( \circ \text{mydev} \) if and only if \( \rho_L(c_L(f(\Theta_a))) = l_j \)
*Components of (simple) pre-measurement*

Let us analyze the components of pre-measurement:

- the mappings \( f \) and \( c_L \) are empirical and need to be **modeled**
- the mapping \( \lambda_L \) is defined by convention
- the mapping \( \rho_L \) is correctly \( \lambda_L^{-1} \) under the hypothesis that instrument is **stable**

and on this basis for any given object \( a \)

- the **transduction** \( L_d = f(\Theta_a) \)
- the **matching** \( L^*_j = c_L(L_d) \)
- the **recognition** \( l_j = \rho_L(L_j) \)

are performed

According to this simple model, pre-measurement results may convey object-related (**objective**) information.

\[
\begin{align*}
\Theta_a \xrightarrow{f} L_d \xrightarrow{c_L} L^*_j \\
& \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
\ & \quad \rho_L \circ c_L \circ f \quad \lambda_L \quad \rho_L \quad l_j
\end{align*}
\]
A model of measurement / 2
(simple measurement)
From pre-measurement to measurement

Pre-measurement results are non-transferable, because private / instrument-specific.

Measurement aims at producing information which is not only object-related, but also subject-independent (“intersubjective”), and therefore public and instrument-independent.

In the tradition of physical measurement this is the task of metrological systems, i.e., measurement standards mutually connected in traceability chains via calibration.
Choosing and identifying temperatures

Some objects / phenomena are chosen whose temperatures are sufficiently stable, so that these temperatures can be identified by

- choosing a set \( \{ \Theta_i^* \} \) of them, each object being a (public) standard of temperature
- assigning an identifier, \( \vartheta_i \), to each temperature, via an injective labeling function

\[
\Theta_i^* = \lambda_\Theta(\vartheta_i)
\]

- inverting \( \lambda_\Theta \) to identify each temperature, via a recognition function

\[
\vartheta_i = \rho_\Theta(\Theta_i^*)
\]

The set \( \{ \Theta_i^* \} \) may be operated as a (public) scale of temperature: let us define \( \{ \Theta_i^* \} := °\text{pub} \)
*Components of (public) scale construction

Let us analyze the components of scale construction:

● the mapping $\lambda_\Theta$ is defined by convention

● the mapping $\rho_\Theta$ is correctly $\lambda_\Theta^{-1}$ under the hypothesis that standards are stable
Calibrating instruments

After having constructed the scale of temperature °pub, the instrument is calibrated by making it interact with the elements of °pub

\[ \Theta_i^* \xrightarrow{f} L_d \xrightarrow{c_L} L_j^* \]

\[ \rho_\Theta(\Theta_i^*) = g(\rho_L(c_L(f(\Theta_i^*)))) \]

The result is the (extensional) definition of the **calibration function** \( g \)

as a set \( \{ \langle l_j, \vartheta_i \rangle \} \) of pairs (private identifier, public identifier)
Components of instrument calibration

Let us analyze the components of instrument calibration:

- *all the components of scale construction*
- the mapping $g$ is correctly the set of pairs $\langle l_j, \vartheta_i \rangle = \{ (\rho_L(c_L(f(\Theta_i^*))), \rho_{\Theta}(\Theta_i^*)) \}$ under the hypothesis that the instrument is **stable**
(simple) measurement

By making the calibrated instrument interact with the quantity to be measured $\Theta_a$, we obtain a measurement

$$\vartheta_i = g(\rho_L(c_L(f(\Theta_a))))$$

The main differences are highlighted...
The structure of (simple) measurement

The result of measurement is
\[ \Theta_a = \vartheta_i \circ pub \] (e.g., \( \Theta_a = 20 \, ^\circ C \))

i.e.,
\[ c_{\Theta}(\Theta_a) = \Theta_i^* \] (e.g., value-of(\( \Theta_a \)) = 20 \, ^\circ C)

i.e.,
\[ m_{\circ pub}(\Theta_a) = \vartheta_i \] (e.g., value-in-\( ^\circ C \)-of(\( \Theta_a \)) = 20)

if and only if
\[ \Theta_i^* = \lambda_\Theta(\Theta_i) = \lambda_\Theta(g(\rho_L(c_L(f(\Theta_a))))) \]
*Components of (simple) measurement

Let us analyze the components of measurement:

- **all the components of pre-measurement**
- **all the components of instrument calibration (and therefore also of scale construction)**

and on this basis for any given object \( a \) and any given calibrated instrument the **transduction** \( L_d = f(\Theta_a) \), the **matching** \( L_j^* = c_L(L_d) \), the **recognition** \( l_j = \rho_L(L_j) \), and the **calibration** \( \vartheta_i = g(l_j) \) are performed.

According to this simple model,

(i) **measurement** = pre-measurement + public scale construction + instrument calibration

(ii) **measurement results** may convey both object-related (**objective**) and subject-independent (**intersubjective**) information.
A model of measurement / 3 – hints

(less simple measurement)
From simple to more realistic measurement

We have assumed so far that
1. the sensor is perfectly selective, i.e., the transduction function $f$ only depends on $\Theta$
2. the quantity to which the value resulting from the measurement is attributed and the quantity that is transduced by the sensor are the same

Both these assumptions may be relaxed, thus making our model more realistic
Non-perfectly selective instruments

In general, the transduced length $L_d$ is caused not only by the temperature $\Theta_a$ but also by other (influence) quantities $Z$

Hence the transduction function is

$$L_d = f_Z(\Theta_a)$$

The limited selectivity of the instrument reduces the objectivity of the information produced by the measurement.
Different intended and effective quantities

According to the International Vocabulary of Metrology (VIM), the measurand is the “quantity intended to be measured” (jcgm.bipm.org/vim/en/2.3.html)

In general, the intended quantity, to which the value is attributed, can be defined as different from the effective quantity, i.e., the transduced quantity

\[ \Theta^E_a \xrightarrow{f} L_d \]

\[ \Theta^I_a \xrightarrow{m_{\text{pub}}} \vartheta_i \]
The structure of (less simple) measurement

The result of measurement is
\[ \Theta_a^I = \vartheta_i \circ \text{pub} \]
i.e.,
\[ c_\Theta(\Theta_a^I) = \Theta_i^* \]
i.e.,
\[ m_{\text{pub}}(\Theta_a^I) = \vartheta_i \]
if and only if
\[ \Theta_i^* = \lambda_\Theta(\vartheta_i) = \lambda_\Theta(g(\rho_L(c_L(f(\Theta_a^E)))))) \]
and (e.g.,)
\[ \Theta_a^E \approx \Theta_a^I \]
Analysis of truth conditions
Truth conditions

Each arrow represents a function, and each function represents a process and is associated with a proposition with a truth value (we say “truth of $f$” as a shorthand for “truth of the proposition that $f(x) = y$”)

Note that
(1) the truth conditions of a function $f: X \rightarrow Y, f(x) = y$, do not depend on the nature of $f$
(2) the truth conditions of a composed function $f_2 \circ f_1$ depend on the truth conditions of $f_1$ and $f_2$, so that only the non-composed functions need to be considered
Truth conditions /2

If these conditions are satisfied, then “\( f(x) = y \)” is true if and only if the process represented by \( f \) produces the entity represented by \( y \) when it receives the entity represented by \( x \) as input.
1. The existence and structure of the domain and range of $\lambda_L$ and $\rho_L$ are non-problematic: 
   \{l_j\} is chosen in correspondence with \{L_j^*\}

2. The existence and structure of the domain and range of $c_L$ are non-problematic: 
   the set \{L_j^*\} is constructed and so it exists

3. The stability of \{L_j^*\} depends on 
   the stability of the instrument
4. The existence and structure of the domain and range of $\lambda_{\Theta}$ and $\rho_{\Theta}$ are non-problematic: 
\{\Theta_i\} is chosen in correspondence with \{\Theta_i^*\}

5. The existence and structure of the domain and range of $c_{\Theta}$ are non-problematic: 
the set \{\Theta_i^*\} is constructed and so it exists

6. The stability of \{\Theta_i^*\} depends on 
the stability of the chosen standards
Analysis of truth conditions

7. The existence and structure of the domain of $f$ is model-dependent:
the transduction effect is assumed to be many-to-one and order-reflecting.

The structure of $\Theta$ is inferred, given the transduction effect, from the structure of $L$.
Analysis of truth conditions /4

Finally, if the instrument is not perfectly selective, so that influence quantities $Z$ are to be considered, and $f_Z$ represents the transduction in conditions $Z$, the definition of $f_Z$ depends on the information available on the influence quantities.
Preliminary conclusions

This schema is a powerful tool for analysing the truth conditions of propositions stating measurement results. In the light of it, four conclusions seem to be justified:

1. a proposition like $\Theta_i^* = \lambda_{\Theta}(\Theta_i) = \lambda_{\Theta}(g(\rho_L(c_L(f_Z(\Theta_a))))))$ can be true

2. the quantity $\Theta_i^*$ is the true value of $\Theta_a$, provided that proposition (1) is true

3. the truth of (1) depends on the stability of the instrument and of the chosen standards and the model $f$ of the transduction

4. propositions (1) and (2), even though model-dependent, are about the world, not about a model
Thank you

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