A pragmatic perspective on measurement

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Outline

- Introduction (to myself and the subject)
- An important problem
- Four standpoints, no (complete) solutions
- A proposal
- A related measurement model
- Concluding remarks, particularly about measurement of non-physical properties
“Those sciences, created almost in our own days, the object of which is man himself, the direct goal of which is the happiness of man, will enjoy a progress no less sure than that of the physical sciences; & this idea so sweet, that our nephews will surpass us in wisdom as in enlightenment, is no longer an illusion. In meditating on the nature of the moral sciences, one cannot help seeing that, as they are based like the physical sciences upon the observation of fact, they must follow the same method, acquire a language equally exact & precise, attaining the same degree of certainty.”

[Condorcet, 1782]
My context (1)

TC1-Education and Training in Measurement and Instrumentation
TC2-Photonics
TC3-Measurement of Force, Mass and Torque
TC4-Measurement of Electrical Quantities
TC5-Hardness Measurement
TC6-Measurement Science
TC7-Traceability in Metrology
TC8-Flow Measurement
TC9-Technical Diagnostics
TC10-Metrological Infrastructures
TC11-Temperature and Thermal Measurements
TC12-Measurements in Biology and Medicine
TC13-Measurements in Biology and Medicine
TC14-Measurement of Geometrical Quantities
TC15-Experimental Mechanics
TC16-Pressure and Vacuum Measurement
TC17-Measurement in Robotics
TC18-Measurement of Human Functions
TC19-Environmental Measurements
TC20-Energy Measurement
TC21-Mathematical Tools for Measurements
TC22-Vibration Measurement
TC23-Metrology in Food and Nutrition
TC24-Chemical Measurements
(JCGM) Joint Committee for Guides in Metrology:

- (BIPM) Int.l Bureau of Weights and Measures
- (IEC) Int.l Electrotechnical Commission
- (IFCC) Int.l Federation of Clinical Chemistry and Laboratory Medicine
- (ILAC) Int.l Laboratory Accreditation Cooperation
- (ISO) Int.l Organization for Standardization
- (IUPAC) Int.l Union of Pure and Applied Chemistry
- (IUPAP) Int.l Union of Pure and Applied Physics
- (OIML) Int.l Organization of Legal Metrology
My context (3)

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the “VIM”

[Image of BIPM logo with the text: International vocabulary of metrology — Basic and general concepts and associated terms (VIM)]

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the “GUM”

[Image of BIPM logo with the text: Evaluation of measurement data — Guide to the expression of uncertainty in measurement]

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gum.html
My background

A basic hypothesis

Being an infrastructural, widespread activity, performed by human beings since millennia, measurement is laden with myths
A few basic concepts and terms:

- **given an object** (phenomenon, event, ...
- **having a property** (attribute, observable, quantity, ...
- **measurement** is a property-related process
- which, applied to the object, produces an **information entity**
- interpreted as a **property value**
- and (with other information) called the **measurement result**
- on the **measurand**, i.e., the property to be measured

Measurement is a property representation process

How is “property” different to “measurand”? And how do these relate to the (psychometric) term “construct”??
Measurement-related models typically assume that:

- there are **general properties** (e.g., length, leadership)
- some general properties can be considered of some objects (*leadership of a person but not of a table]*
- a general property of an object is an **individual property** of that object (*length of a given table, leadership of a given person*)
- a measurement problem is about a general property (*I would like to measure leadership*)
- measurement is applied to individual properties (*I am measuring the leadership of this person*)
- a general property is characterized by a set of property values (*positive real numbers for length*)
- an individual property is represented by a property value (*the length of this table is 2.34 m*)
These assumptions lead to a functional model of the involved entities where:

- general properties are described as functions,
- whose domain is a set of objects
- and whose range is a set of individual properties

\[ p_{gen} : \{\text{objects}\} \rightarrow \{p_{ind}\} \]

(your leadership is modeled as leadership(you), so that the fact that leadership is not considered of tables, i.e., leadership(this table) is wrong, is modeled as the hypothesis that tables do not belong to the domain of the function leadership)

Such functions describe empirical facts

Measurement is aimed at representing individual properties

(this is written with the “=” symbol, length(this table) = 2.34 m, properly meaning “is represented by”, not “is equal to”)
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*A pragmatic perspective on measurement*
What is the problem?

The previous description applies not only to measurement, but generically to processes of assignment of property values, (property) evaluations for short (i.e. it gives necessary but not sufficient conditions to define ‘measurement’)

How is measurement characterized as a specific kind of evaluation?

evaluations

measurements
What is the importance?

- From an **epistemic** point of view: measurement results are considered conveying “reliable” information on properties: **what is the source of such reliability?**

- From a **pragmatic** point of view: it is socially accepted that obtaining measurement results requires employing some resources: **under what conditions is such acceptation justified?**

Say more about what you mean by “reliability”.
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A pragmatic perspective on measurement
Is it an already solved problem?

Several standpoints...

Measurement is:

- S1: process producing Euclidean ‘measures’
- S2: physical transducer operation
- S3: morphic representation
- S4: decision making support
S1. Measurement as a process producing Euclidean ‘measures’

“A magnitude is a part of a magnitude, the less of the greater, when it measures the greater; the greater is a multiple of the less when it is measured by the less; a ratio is a sort of relation in respect of size between two magnitudes of the same kind.”

[Euclid, Elements, Book V, definitions 1-3]

→ A property is measurable because it can be represented by property values of the form \( n \cdot u \), where \( n \) is an integer number and \( u \) is a “unit” property
S1. Significance / benefits

- This standpoint is the basis of the classical concept of **quantity** (a quantity is a property representable as multiple of a unit), and therefore of quantity calculus / dimensional analysis, where quantities are represented, in Maxwell’s notation, as:

  \[ q = \{q\}[q] \]

  - \( q \): quantity to be represented
  - \( \{q\} \): numerical quantity value
  - \([q]\): unit

- The International System of Quantities (ISQ), and then the International System of Units (SI), are based on quantities in the Euclidean sense
S1. Objections

- This standpoint does not give any justification of the claimed reliability of measurement results ("according to my experience, I can see that this object is 1,2 m long" expresses in fact a ratio of two "magnitudes"; nevertheless, this is hardly acceptable as a measurement result)

- It is today customarily accepted that less-than-ratio properties (e.g., ordinal) can be measurable

  This characterization gives neither necessary nor sufficient conditions of measurability
S2. Measurement as the process performed by a physical transducer

The “geometrical paradigm” has been successfully exported to the physical world and embedded in a metrological infrastructure.

→ **A property is measurable because** it is the input signal of a properly calibrated and operated instrument realizing a physical transduction effect.
S2. Significance / benefits

- This standpoint emphasizes that measurability has to do with the way the information on the property is acquired, not (only) the way it is represented.

  - This characterization could give sufficient conditions of measurability.
S2. Objections

- Although effective in measurement of physical properties, this standpoint is useless if the aim is to characterize the measurability of non-physical properties

  This characterization does not give necessary conditions of measurability
S3. Measurement as a morphic representation of properties

“Measurement is the assignment of numerals to objects or events according to rule, any rule.”

[Stevens 1959]

As the outcome of a critical analysis on the possibilities of applying measurement in social sciences, measurement has been axiomatized as a morphic mapping from properties to property values (e.g., if $p(a) < p(b)$ then $m_p(a) < m_p(b)$)

→ A property is measurable because it can be mapped to a set of property values and the mapping is a morphism
S3. Significance / benefits

- This standpoint has been very fruitful in terms of its theoretical consequences, as it is the basis of the so called “representational theories of measurement”: multiple “measurement scales” are identified (e.g., nominal, ordinal, …), and for each of them a representation theorem (what conditions are required for a morphic mapping to be defined) and a uniqueness theorem (what conditions constrain the values assigned by the morphic mapping) are given.

- This characterization gives a parametric set of (plausibly) necessary conditions of measurability.
S3. Objections

- As for the first objection to S1, this standpoint does not give any justification of the claimed reliability of measurement results ("according to my experience, I can see that the object \( a \) is shorter than \( b \), and therefore the length value I have assigned to \( a \) is less than the length value assigned to \( b \); nevertheless, this is hardly acceptable as a measurement result)

- This characterization does not give sufficient conditions of measurability
S4. Measurement as a process supporting decision making

A measurement result is the “symbolic representation of event, state or attribute, to aid in the making of a decision”

[Nicholas, White 2001]

→ A property is measurable because its values, as obtained by means of measurement, are useful in decision making
This standpoint emphasizes that, as any production process, measurement should be justified in terms of the usefulness of the results it produces.

This characterization might give a (very loose) necessary condition of measurability.
S4. Objections

- Is it really appropriate to characterize measurement as any “useful” (?) “symbolic representation” (?)?

- This characterization does not give actual conditions of measurability
An open problem, then

Measurement is:

- S1: process producing Euclidean ‘measures’
- S2: physical transducer operation
- S3: morphic representation
- S4: decision making support

Several standpoints, but none of them fully appropriate to characterize a concept of measurement general enough to encompass non-physical properties but specific enough to exclude generic evaluations.
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A pragmatic perspective on measurement
Beyond S1 – S4?

The hypothesis that measurement is a property evaluation whose results convey reliable information on the measurand is not related to:

- the nature of the object under measurement or of the measurand (both physical and non-physical properties should be in principle measurable)
- the algebraic structure of the set of property values (not only Euclidean quantities should be in principle measurable)

What is the epistemic source of such reliability which pragmatically justifies employing some resources to obtain measurement results?
Conceptual proposal

This reliability is justified in terms of two general features expected for measurement results, which in principle are supposed to convey information:

- **specific to the measurand**, and independent of any other property of the object or the surrounding environment, which includes both the measuring system and the subject who is measuring;
- **interpretable in the same way by different users** in different places and times, and therefore expressed in a form independent of the specific context and only referring to entities which are universally accessible.

In the phrase “specific to the measurand, and independent of any other property of the object or the surrounding environment”, would it be OK to add a few words to say the following: “specific to the measurand, and independent of the measurement of any other property of the object or the surrounding environment”.

Lexical proposal

The supposition that the information conveyed by measurement results

- is specific to the measurand, and therefore to the object of measurement, is a requirement of **objectivity**
- is universally interpretable, and therefore is the same for different individuals, is a requirement of **intersubjectivity**

Accordingly, objectivity and intersubjectivity are independent features

(an evaluation might be objective but not intersubjective, or intersubjective but not objective)

Give some examples of objectivity and intersubjectivity.
Pragmatic proposal

Neither objectivity nor intersubjectivity of measurement results are Boolean (i.e., yes-no) features.

Measurement results have an overall degree of “quality”, customarily expressed in terms of their (un)certainty and related to their objectivity and intersubjectivity.

Hence, if the usefulness of measurement results has to be taken into account then their uncertainty must be less than a target measurement uncertainty (“measurement uncertainty specified as an upper limit and decided on the basis of the intended use of measurement results”), i.e., they must be sufficiently objective and sufficiently intersubjective.

Perhaps, after this, review S1-S4 and mention important differences/similarities.
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A pragmatic perspective on measurement
A simple model of measurement

Measurement:

\[ p \xrightarrow{m} v \]

is a structured, empirical + representational, process, based on the possibility of transducing \( p \) to a property \( \overline{p} \) (the “indication”):

\[ p \xrightarrow{\tau_p} \overline{p} \]

and then mapping \( \overline{p} \) to a value \( \overline{v} \) (the “indication value”):

\[ p \xrightarrow{\tau_p} \overline{p} \xrightarrow{} \overline{v} \]

where the empirical mapping \( \tau_p \) is required to be calibrated.
Calibration

Let us suppose that a set of standards is available such that each of them:

- realizes a reference property $r$
- is associated with a given property value $v_r$
- is transduced to an indication $\overline{p}$, and then associated with an indication value $\overline{v}$

Then a mapping $\tau_v$ ("calibration function") can be construed:
Measurement (simplest, ideal version)

Under the hypothesis that:

- the calibration function can be inverted
- the transduction is stable (the function did not change)

the measurement $p \rightarrow v$ is performed by:

1. transducing the measurand $p$ to an indication $\bar{p}$
2. mapping the indication $\bar{p}$ to the indication value $\bar{v}$
3. mapping the indication value $\bar{v}$ to a measurand value $v$
It might be acknowledged that:

\[ p \xrightarrow{\tau_p} \bar{p} \]

the transducer is not perfectly stable, because it is sensitive to some influence properties other than the measurand (i.e., the transducer does not behave as an ideal filter) so that the indication \( \bar{p} \) depends not only on the measurand \( p \) but also on such other properties

**This reduces the objectivity of measurement:** measuring systems are designed to minimize such effects and therefore to maximize objectivity.
It might be acknowledged that:

the standards are not perfectly stable, and for all non-primary standards the information on the reference property value is uncertain so that the reference properties $r$ are not mapped to a single property value $v_r$

This reduces the intersubjectivity of measurement: standards are designed to minimize such effects and therefore to maximize intersubjectivity
Calibration (more realistic version)

Since:

then the mapping \( \tau_v \) becomes a “calibration strip”: 
As a consequence, even when a single transduction is performed:

$$\tau_p$$

$$\tau_v^{-1}$$

Pragmatically: measurement uncertainty should be less than target uncertainty
In a more general case, the measurand $p$ might be not
the input property of a transducer, but is dependent,
through a given function $f$, on one or more properties $p_i$
that can be transduced (or whose values are somehow
known)

The previous process becomes a component of the whole ("indirect") measurement process:

$$f(p_1, p_2, ...)$$

$$f(v_1, v_2, ...)$$

Hence, the uncertainty on the values $v_i$
must be "propagated" through $f$ to compute the
uncertainty of $v$
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The problem of measurand

Measurement bridges two worlds...

\[
\begin{align*}
p & \xrightarrow{\tau_p} p_i \\
\downarrow \tau_v^{-1} & \quad \downarrow \tau_v \\
v & \xleftarrow{} v_i
\end{align*}
\]

empirical world

information world

The last step is to interpret the obtained property value, \( v \), as a value of the measurand, i.e., the property intended to be measured:

is the property which I have measured actually the property which I intended to measure?
‘Definitional uncertainty’

“component of measurement uncertainty resulting from the finite amount of detail in the definition of a measurand”

Note: “definitional uncertainty is the practical minimum measurement uncertainty achievable in any measurement of a given measurand”
A source of uncertainty comes from the "correspondence" between the problem/process and its model.
Measurement of non-physical properties

(just to trigger the discussion)

Nothing in this presentation implies the physical nature of measurands; hence this analysis and its conclusions seem to be applicable also to non-physical properties

Nevertheless, the some differences (typically) remain
Differences...

1. Physical quantities are mutually related by physical laws; this allows:
   • minimizing primitive ("purely operational") concepts
   • cross-validating measurand definitions
   • cross-checking measurements results
2. A global metrological infrastructure is well established for physical quantities
3. The measurement of physical properties is a purely descriptive process
4. Physical properties have been measured since millennia

Slide 49: What would be a likely alternative to "descriptive"?
A pragmatic perspective on measurement

Thank you for the kind attention

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